Generalizations of the Simple Mover-stayer Model, Association Models and Quasi-symmetry Models, in the Analysis of Brand-switching Data and Other Cross-classified Data

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The simple mover-stayer model considered here was introduced by Blumen, Kogan, and McCarthy (1955). The statistical procedures applied in their 1955 paper for analyzing this model required correction (see Goodman 1961), and correct methods were presented in a series of articles by Goodman (1961, 1963, 1964, 1968, 1969, 1972). The simple mover-stayer model can be viewed, in some sense, as equivalent to the model of quasi-independence in a two-way contingency table for the entries off the main-diagonal (see, e.g., Goodman 1963, 1969, 1972), and the more recent forms of association models and quasi-symmetry models (when applied to a two-way contingency table for the entries off the main-diagonal) can be viewed as generalizations of these models (see, e.g., Goodman 1979, 1985, 1986, 1991a,b). The simple mover-stayer model (or corresponding quasi-independence model) and the generalizations noted above will be applied to the 16 x 16 table (Table 1) obtained from the 15 x 15 brand-switching French data for 1989 for the 15 auto makes, by introducing a 16-th column and 16-th row (viz., column N, which includes purchases of other makes and cases where there is no information about make purchased, and row N, which includes previous ownership of other makes or of no car at all and cases where there is no information about previous ownership). Tables 1 to 4 pertain to the 16 x 16 table, and Tables 5 to 7 pertain to the original 15 x 15 table.

The independence model O applied to the 16 x 16 table can be tested as usual, with 15 x 15 = 225 d.f., and the quasi-independence model O' applied to the entries in this table off the main-diagonal can be tested with 225 - 16 = 209 d.f. The uniform association model U', the association model RC_H' in which the row-column scores are homogeneous, and the symmetric association model QS', are all models that are applied here to the entries off the main-diagonal, and these models can be tested with 209 - 1 = 208 d.f., 15 x 14 - 16 = 194 d.f., and 15 x 14 / 2 = 105 d.f., respectively (see, e.g., Goodman 1979, 1985, 1991a). Table 2 gives the results obtained for these models, and corresponding results are presented in Table 3 for the results obtained when the association models RC_1', RC_2', RC_3', and RC_4' (one-, two-, three-, and four-component models) are applied to the entries in the 16 x 16 table off the main-diagonal, with 14 x 14 - 16 = 180 d.f., 13 x 13 - 16 = 153 d.f., 12 x 12 - 16 = 128 d.f., and 11 x 11 - 16 = 105 d.f., respectively (see, e.g., Goodman 1985, 1986). The quasi-independence model O' (or the corresponding simple mover-stayer model) explains a large part of the association in the 16 x 16 table (see Table 2.2 and 3.2), but the model is not congruent with the data if the usual statistical criteria are applied. The RC_4' association model (with 105 d.f.) is congruent with the data using the usual criteria, and the QS' symmetric association model (with 105 d.f.) is not. (The QS' model is equivalent to the usual quasi-symmetry model.) Table 4 gives the maximum-likelihood estimates of the parameters in the simple mover-stayer model (corresponding to quasi-independence model O') and also some other related quantities. The *adjusted* column marginal proportion for, say, auto-make Re is .259 (see Table 4.1), which is the estimated probability that a potential mover (i.e., a potential switcher) will buy auto-make Re (regardless of previous auto-make); and the corresponding index of Wpersistence is .49 (see Table 4.2), which is the estimated proportion of stayers (i.e., hard-core auto-make-loyal owners) among those whose previous auto-make is Re, estimated under the simple mover-stayer model (see, e.g., Goodman 1961, 1969, 1972). The estimates of the indices of persistence in Table 4.2, and the other estimates presented there, which were calculated under the simple mover-stayer model (or the corresponding quasi-independence model O'), will change when this model is replaced by one of the generalizations (e.g., association

model RC_4' or symmetric association model QS'); and the probability that a potential mover (i.e., a potential switcher) will buy a given auto-make, which does not depend on previous auto-make under the simple mover-stayer model, will change to a probability that does depend on previous auto-make when this model is replaced by one of the generalizations. The results for the generalizations will be presented in a separate report along with appropriate graphical displays.

As in Tables 2 and 3 for the 16 x 16 table, we give in Tables 5 and 6 the corresponding results for the 15 x 15 table (deleting the 16-th row and column). From Tables 5 and 6, we see that the quasi-independence model (or the corresponding simple mover-stayer model) explains a large part of the association in the 15 x 15 table, but the model is not congruent with the data if the usual statistical criteria are applied. The RC_4' association model (with 85 d.f.) fits the data very well, and the RC_3' association model (with 106 d.f.) is also congruent with the data using the usual criteria. The usual quasi-symmetry model QS' (with 91 d.f.) is not congruent with the data, but a modified quasi-symmetry model QS" (with 86 d.f.) is. The modification of the QS' model is obtained by allowing five exceptions to symmetric association (see, e.g., Goodman 1991b). The exceptions in the 15 x 15 table are for entries C - Se, VW - Ro, VW - P, VW - Re, and M - B. (It is important to note that the selection of these entries was data-based.)

Tables corresponding to Table 4 can also be presented for the 15 x 15 table, based on the simple mover-stayer model and on the generalizations noted above. Association model RC_3' or RC_4' also lead to the determination of the three or four dimensions that describe the relationships among the auto-makes (with respect to brand-switching); and when the row x column interaction parameters in the QS" model are appropriately defined, the estimates of these parameters also provide a way of ordering the auto-makes. All of this will be presented in a separate report along with appropriate graphical displays. As an indication of one of the kinds of results included in the separate report we provide here Table 7, the row x column estimated interactions (when appropriately defined) under the QS" model. (In order to facilitate understanding of this kind of table and provide somewhat simpler results, the results are presented here for the QS" model applied to the 13 x 13 table obtained when auto-makes B and L are omitted.)

Table 1:The 16 x 16 table of counts for 15 auto make categories and one residual category (viz., N) that includes other makes and no information

Previous Make	Newly Purchased Make															
	A	B	C	Fi	Fo	G	L	M	P	Re	Ro	Sa	Se	VW	Vo	N
A	97	5	19	19	14	7	0	3	27	35	5	1	6	17	4	9
B	4	163	20	9	14	6	0	19	39	40	6	3	4	29	4	14
C	6	13	1811	136	98	67	21	15	464	477	29	1	28	90	10	45
Fi	11	5	69	526	50	49	12	3	134	148	27	1	17	82	5	26
Fo	4	10	45	53	696	76	8	8	121	146	20	1	23	60	7	34
G	2	7	20	23	50	362	7	4	68	80	7	0	12	39	4	25
L	0	0	12	13	11	12	68	0	25	16	4	0	3	7	0	9
M	2	17	3	3	5	2	0	136	4	10	3	1	1	5	3	15
P	13	23	273	164	195	168	34	23	2928	728	47	2	60	199	20	85
Re	22	37	353	334	308	254	40	33	896	4861	78	4	106	310	24	115
Ro	2	4	18	25	30	3	6	1	30	45	115	0	6	40	2	10
Sa	1	2	0	1	0	0	0	2	0	0	0	10	0	2	1	1
Se	2	0	13	10	10	11	0	0	10	29	3	0	36	13	0	0
VW	6	21	50	51	65	59	2	10	177	115	13	3	25	772	9	39
Vo	1	3	11	5	4	6	0	3	9	17	3	2	4	18	78	5
N	36	57	467	275	289	209	35	53	907	1190	80	7	63	303	33	423

Table 2.1: Association models applied to 16 x 16 table

Models	Degrees of Freedom	Goodness-of-fit chi-square	Likelihood-ratio chi-square
Independence model O	225	61110.76	22817.55
Quasi-independence model O'	209	1245.25	859.29
Uniform association model U'	208	664.79	588.23
Homogeneous RC association model RC_H'	194	588.23	549.41
Symmetric association model QS'	105	243.75	231.66

Table 2.2: Analysis of Association (ANOAS) in 16 x 16 Table: Components in O Model

Components	Models Used	Degrees of Freedom	Likelihood-ratio chi-square	Component percentage
Non-independence on main-diagonal in O' model	O-O'	16	21958.26	96.2%
Symmetric association in QS' model	O'-QS'	104	627.63	2.8%
Asymmetric association	QS'	105	231.66	1.0%
Total effects	O	225	22817.55	100.0%

Table 2.3: ANOAS in 16 x 16 Table of Main-Diagonal: Components in O' Model

Components	Models Used	Degrees of Freedom	Likelihood-ratio chi-square	Component percentage
Association in U' model	O'-U'	1	271.06	31.5%
Unequal spacing of row-column scores in RC'_H model	U'-RC'_H	14	38.72	4.6%
Symmetric association not in RC'_H model	RC'_H-QS'	89	317.85	37.0%
Asymmetric	QS'	105	231.66	27.0%
Total effects		209	859.29	100.0%

Table 3.1: Association Models Applied to 16 x 16 Table

Models	Degrees of Freedom	Goodness-of-fit chi-square	Likelihood-ratio chi-square
Independence model O	225	61110.76	22817.55
Quasi-independence model O'	209	1245.25	859.29
Association model RC'_1	180	410.63	421.58
Association model RC'_2	153	250.66	260.13
Association model RC'_3	128	160.67	167.91
Association model RC'_4	105	112.83	114.37

Table 3.2: Analysis of Association (ANOAS) in 16 x 16 Table: Components in O Model

Components	Models Used	Degrees of Freedom	Likelihood-ratio chi-square	Component percentage
Non-independence on main-diagonal in O' model	O-O	16	21958.26	96.2%
Association off main-diagonal explained in RC'_4 model	O'-RC'_4	104	744.92	3.3%
Association unexplained in RC'_4 model	RC'_4	105	114.37	0.5%
Total effects	O	225	22817.55	100.0%

Table 3.3: ANOAS in 16 x 16 Table off Main-Diagonal: Components in O' Model

Components	Models Used	Degrees of Freedom	Likelihood-ratio chi-square	Component percentage
Association in RC'_1 model	O'-RC'_1	291	437.71	50.9%
Second component in RC'_2 model	RC'_1-RC'_2	27	161.45	18.8%
Third component in RC'_3 model	RC'_2-RC'_3	25	92.22	10.7%
Fourth component in RC'_4 model	RC'_3-RC'_4	23	53.54	6.2%
Association unexplained in RC'_4 model	RC'_4	105	114.37	13.3%
Total effects	O'	209	859.29	100.0%

Table 4.1: Marginal Percentages for 16 x 16 Table. Gross, Net and Adjusted Marginals for Quasi-Independence Model O' off Main-Diagonal

Auto Make	Column Marginal			Row Marginal		
	Gross	Net	Adjusted	Net	Gross	Adjusted
A	0.8	0.8	0.7	1.0	1.2	1.1
B	1.4	1.5	1.3	1.4	1.5	1.4
C	11.9	10.0	9.7	12.4	11.0	10.5
Fi	6.2	8.2	7.4	4.4	4.7	4.4
Fo	6.9	8.3	7.6	4.9	4.5	4.2
G	4.8	6.8	6.0	2.7	2.5	2.3
L	0.9	1.2	1.1	0.7	0.8	0.7
M	1.2	1.3	1.1	0.8	0.5	0.5
P	21.8	21.3	22.1	18.5	14.9	16.5
Re	29.6	22.5	25.9	29.0	21.3	24.9
Ro	1.6	2.4	2.1	1.3	1.6	1.4
Sa	0.1	0.2	0.2	0.1	0.1	0.1
Se	1.5	2.6	2.3	0.5	0.7	0.7
Vw	7.4	8.9	8.0	5.3	4.7	4.4
Vo	0.8	0.9	0.8	0.6	0.7	0.6
N	3.2	3.2	3.7	16.5	29.2	26.3

Table 4.2: Indices of Persistence and a Ratio Index for Quasi-Independence Model O' off Main-Diagonal

Auto Make	Row-Persistence	Column-Persistence	Stayer-Ratio
A	.36	.46	78.6
B	.43	.44	58.2
C	.50	.52	11.2
Fi	.41	.29	10.3
Fe	.49	.35	13.8
G	.48	.26	16.2
L	.37	.29	57.1
M	.64	.43	161.4
P	.47	.40	5.1
Re	.49	.48	4.8
Ro	.33	.25	24.3
Sa	.50	.28	606.2
Se	.25	.09	15.3
VW	.51	.36	13.7
Vo	.46	.38	106.0
N	.06	.31	2.7

Table 5.1: Association Models Applied to 15 x 15 Table

Models	Degrees of Freedom	Goodness-of-fit chi-square	Likelihood-ratio chi-square
Independence model O	196	61027.96	22219.69
Quasi-independence model O'	181	1148.15	706.33
Uniform association model U'	180	446.63	417.74
Homogeneous RC association model RC'_H	167	377.08	375.57
Symmetric association model QS'	91	125.92	129.82
Modified quasi-symmetry QS"	86	86.11	89.49

Table 5.2: Analysis of Association (ANOAS) in 15 x 15 Table: Components in O Model

Components	Models Used	Degrees of Freedom	Likelihood-ratio chi-square	Component percentage
Non-independence on main-diagonal in O' model	O-O	15	21513.36	96.8%
Symmetric association in QS' model	O'-QS'	95	616.84	2.8%
Association unexplained in QS" model	QS'	86	89.49	1.4%
Total effects	O	196	22219.69	100.0%

Table 5.3: ANOAS in 15 x 15 Table of Main-Diagonal: Components in O' Model

Components	Models Used	Degrees of Freedom	Likelihood-ratio chi-square	Component percentage
Association in U' model	O'-U'	1	288.59	40.9%
Unequal spacing of row-column scores in RC'_H model	U'-RC'_H	13	42.17	6.0%
Symmetric association not in RC'_H model	RC'_H-QS'	76	245.75	34.8%
Asymmetric association in QS" model	QS'-QS"	5	40.33	5.7%
Association unexplained in QS" model	QS"	86	89.49	12.7%
Total effects		181	706.33	100.0%

Table 6.1: Association Models Applied to 15 x 15 Table

Models	Degrees of Freedom	Goodness-of-fit chi-square	Likelihood-ratio chi-square
Independence model O	196	61027.96	22219.69
Quasi-independence model O'	181	1148.15	706.33
Association model RC'_1	154	315.96	326.24
Association model RC'_2	129	192.41	203.20
Association model RC'_3	106	111.60	117.23
Association model RC'_4	85	69.71	70.75

Table 6.2: Analysis of Association (ANOAS) in 15 x 15 Table: Components in O Model

Components	Models Used	Degrees of Freedom	Likelihood-ratio chi-square	Component percentage
Non-independence on main-diagonal in O' model	O-O'	15	21513.36	96.8%
Association off main-diagonal explained in RC'_4 model	O'-RC'_4	96	635.58	2.9%
Association unexplained in RC'_4 model	RC'_4	85	70.75	0.3%
Total effects	O	196	22219.69	100.0%

Table 6.3: ANOAS in 15 x 15 Table off Main-Diagonal: Components in O' Model

Components	Models Used	Degrees of Freedom	Likelihood-ratio chi-square	Component percentage
Association in RC'_1 model	O'-RC'_1	27	380.09	53.8%
Second component in RC'_2 model	RC'_1-RC'_2	25	123.04	17.4%
Third component in RC'_3 model	RC'_2-RC'_3	23	85.97	12.2%
Fourth component in RC'_4 model	RC'_3-RC'_4	21	46.48	6.6%
Association unexplained in RC'_4 model	RC'_4	85	70.75	10.0%
Total effects	O'	209	706.33	100.0%

Table 7: Row x Column Interactions in Modified Quasi-Symmetry Model QS" Applied to 13 x 13 Table Obtained from 15 x 15 Table with Deleted B and L Auto-Makes

	G	Ro	Se	C	P	Re	Fo	Fi	VW	A	Vo	M	Sa
G	--	18.7	19.5	12.9	13.2	12.9	13.2	12.0	11.4	10.0	9.6	9.1	0.0
Ro		--	18.5	12.4	12.2	12.0	12.4	11.7	11.2	9.9	9.0	8.7	0.0
Se			--	12.7	11.9	11.9	11.9	11.0	10.4	9.7	8.6	7.3	0.0
C			11.8	--	6.7	6.2	5.6	5.3	3.9	3.5	2.6	2.2	0.0
P					--	6.0	5.6	4.9	4.0	3.1	2.2	1.9	0.0
Re						--	5.3	4.7	3.7	2.9	1.9	1.7	0.0
Fo							--	4.4	3.7	2.8	1.8	1.9	0.0
Fi								--	3.1	2.7	1.1	0.4	0.0
VW		10.2			4.4	3.4			--	1.4	1.1	0.4	0.0
A										--	0.5	0.2	0.0
Vo											--	0.0	0.0
M												--	0.0
Sa													--