The Structure of Inter-brand Competition

Manohar U. Kalwani, P.K. Kannan, Byunghoon Lim

Introduction

An understanding of the competitive structure of a market is important to the formulation of effective marketing strategies. Decisions such as whether or not to introduce a new product or the repositioning of an existing product depend to a significant extent upon knowledge of how the market is partitioned. A market partition (or a submarket) is comprised of brands that compete most strongly with one another. Firms often desire to have a brand in each of the major partitions of a market and to avoid unnecessary duplication of brands. If a marketing manager can identify a submarket where the firm does not already have a brand, it can consider development of a new brand for that submarket to gain additional sales or profit; or if it can identify the submarket where the firm has two similar brands competing against and cannibalizing each other, it can decide to discontinue one of the brands. In addition to new product introductions and product repositioning strategies, we suggest that advertising, pricing, and product reformulation decisions also depend in part on the information derived from the analysis of the competitive structure of a given market.

In addition to the above considerations, a number of useful marketing models that are commonly used to study the consumer purchase behavior in a market are applicable to only single partitioned markets. For example, when a market is structured into multiple partitions, the use of a multinomial-Dirichlet model at the aggregate level is no longer valid and may result in a poor fit and unreliable predictions (Goodhardt, Ehrenberg and Chatfield 1984). In such cases, it is necessary to identify a priori the appropriate partitions in the market and apply brand choice models to individual partitions within the market. Thus, the identification of competitive structure of a market needs to precede many detailed analyses.

Many aggregate approaches have been used in prior research to characterize competition in a market. They include methods based on brand switching (e.g., Kalwani and Morrison 1977; Urban, Johnson and Hauser [UJH hereafter] 1984), cross-elasticity of demand (e.g., Allenby 1989), inter-purchase times (e.g., Fraser and Bradford 1983), in-use substitution (e.g., Day, Shocker, and Srivastava 1979), and measures based on consumer choice models (e.g., Vilcassim 1989). The use of brand switching data based methods has been, by far, the most popular because of their simplicity of modeling, ease of interpretability, and ease of data availability. The approaches based on brand switching data can be further classified as "exploratory" in the sense of uncovering structure from data (e.g., Rao and Sabavala 1981; Grover and Srinivasan 1987), or as "confirmatory" methods which test for specific structures (e.g., Kalwani and Morrison 1977; UJH 1984).

In this paper, we apply selected cluster and partitioning approaches to the 198D French automobile market brand switching data with a view to understand the structure of market competition. We perform cluster analysis on brand interaction indices (National Purchase Diary [NPD] 1984) and proximity measures (Rao and Sabavala 1981) to identify alternate market partitioning hypotheses. We then employ confirmatory techniques such as the multinomial-Dirichlet approach (Kalwani and Morrison 1977) and the "forced switching" approach (UJH 1984) to test the market partitioning hypotheses identified from cluster analysis. The objective of our exercise is to examine and explain the degree of conformance of the results obtained using the different approaches with a view to forming an understanding of the nature of the inter-brand competition.

The paper is organized as follows: in the next section, we provide an overview of the analytical procedures that we apply to the French automobile data. Then, we present the findings from the applications of these procedures and discuss the conformance among them. Next, we present our summary view of the inter-brand competition in the market. We conclude with a discussion on the comparative merits of the different approaches and a framework for further research.

An Overview of Market Partitioning Approaches

In this section we present a brief overview of the different approaches that we use in analyzing the French automobile data. We first describe the cluster analysis procedures that we use to formulate tentative partitioning hypotheses and then the UJH and multinomial-Dirichlet model analyses that we use to test the hypothesized competitive market structures.

Cluster Analysis

Cluster analysis is a common exploratory technique that may be used to group brands into partitions that reflect the strength of competition between brands. It is relatively easy and straightforward to apply. The most important factor that determines the "structure" that is uncovered from the data is the inter-brand substitutability measure, which is constructed from the brand switching data. The substitutability measure reflects the strength of competition between brands and there are alternate ways of estimating it. We discuss two common approaches below:

Index of Interaction: The NPD market research firm has proposed an index of interaction that compares the patterns of the purchasing of the other brands in the market by a brand's buyers versus the patterns of other brand purchasing in the total market (see NPD 1975). Let b be the number of brands in the market. Further, let,

m_i = the market share of brand i (i = 1,...,b),
n_i = the total number of purchases of all brands by buyers of brand i (i = 1,...,b), and
n_{ij} = the total number of purchases of brand j by buyers of brand i (i = 1,...,b).

Then, the fraction of brand j purchases to the total purchases of other brands (j = 1,2,...,b; $j \neq i$) by buyers of brand i is given by:

$$f_{ij} = \frac{n_{ij}}{n_i - n_{ii}} \tag{1}$$

The market share of brand j in the absence of brand i from the market, m_{ij}, is given by:

$$m_{ij} = \frac{m_j}{1 - m_i} \tag{2}$$

The index of interaction, I_{ij}, between brand i and brand j is given by:

$$I_{ij} = \frac{f_{ij}}{m_{ij}} \text{ x } 100. \tag{3}$$

Thus, the index of interaction, I_{ij}, compares the share of purchases devoted by brand i's buyers to brand j on occasions when they do not buy brand i to the market share of brand j in the absence of brand i from the market. Thus, the indices of brand interaction help identify competitive sets, comprised of brands that are most closely identified by consumers as acceptable substitutes. The index of interaction, I_{ij}, takes the value 100 when brand i's buyers are no more likely to buy brand j than the rest of the market in the absence of brand i from the choice set. If I_{ij} exceeds 100 by a significant amount, it indicates that brand j belongs to the competitive set of brand i, and the higher the value, of course, the stronger is this competitive relationship. Similarly, values less than 100 indicate the brands belong in different competitive sets. Thus, using the index of interaction as a similarity index, we use cluster analysis to identify competitive sets or partitions of the market where within-partition competition is much stronger than across-partition competition.

Proximity Measure: Rao and Sabavala (1981) propose a clustering procedure for inferring hierarchical market partitions from brand switching data for a group of consumers who are assumed homogeneous in their hierarchical choice process. The similarity index used for the clustering procedure can be interpreted as the ratio of the actual number of consumers switching from brand *i* to brand *j* to the expected number switching from *i* to *j* under the assumption of a zero-order choice process and the choice probabilities given by the market shares of brands *i* and j, respectively. Formally, let

n_{ij} = number of consumers who switched from brand *i* on the first choice occasion to brand *j* on the second,
$n_{i.}$ = number of consumers who purchased brand *i* on the first choice occasion,
$n_{.j}$ = number of consumers who purchased brand *j* on the second choice occasion, and
$n_{..}$ = number of consumers in the homogeneous population.

Then, the proximity measure, F_{ij}, is defined as:

$$F_{ij} = \frac{n_{ij}}{(n_{i.}n_{.j} / n_{..})} \tag{4}$$

The proximity measure is in spirit similar to the index of interaction. They each will tend to be high when a brand's customers are more likely to switch to a particular brand relative to their propensity to switch to the other brands and they will each tend to be low in the reverse situation.

The "Forced Switching" Approach

The confirmatory approach of Urban, Johnson, and Hauser (1984) defines the null hypothesis of no structure as a market in which a brand draws switchers from other brands in proportion to the market shares of the other brands. A market is structured into partitions, if, when a brand is deleted from a partition, its former consumers are more likely to buy again in that partition than would be predicted by market shares. The structure to be tested is operationalized as an alternate hypothesis. Thus, the testing procedure essentially involves comparing the actual switching levels to the switching levels predicted by market shares. Although the UJH procedure has been operationalized in a "forced switching" context, where consumers of a brand are forced to switch to other brands through the deletion of their favorite brand from the market (an incomplete contingency table where repeat purchase of a brand is not allowed), it can be applied to brand switching data (a complete contingency table where repeat purchases are allowed) through slight modifications as shown below:

Let s = a set of brands, called submarket *s* or partition *s*,
n_i = number of consumers who chose brand *i* on the first occasion and then switched out of it for the next purchase occasion,
$n_i(j)$ = the number of consumers out of n_i who formerly chose brand *i*, but now choose brand *j* on the second occasion,
$n_i(s)$ = the number of consumers out of n_i who formerly chose brand *i*, but now choose a brand from partition *s*,
$P_i(j)$ = the overall market share of brand *j*, with brand *i* being no longer a choice (for switchers, who switch out of brand i, brand *i* is no longer a choice),
$P_i(s)$ = the market share of partition *s*, brand *i* being no longer a choice,
m_j = market share of brand *j* (m_j is estimated from the brand switching data using the first choice data).

Given the above definition it follows that:

$$n(s) = \sum_{i \in s} n_i(s) \tag{5}$$

$$n^* = \sum_{s \in T} n(s) \tag{6}$$

where T = total market,

$$P_i(j) = \frac{m_j}{1 - m_i} \tag{7}$$

and

$$P_i(s) = \sum_{j \in s,\ j \neq i} P_i(j) \tag{8}$$

The key result of the UJH approach to testing competitive market structures is that, under the null hypothesis of no structure, the observed values *n(s)* and *n** follow normal distributions as below:

$$n(s) \sim N\left(\sum_{i \in s} n_i P_i(s),\ \ \sum_{i \in s} n_i P_i(s)\,(1 - P_i(s)) \right) \tag{9}$$

and

$$n^* \sim N\left(\sum_{s \in T} \sum_{i \in s} n_i P_i(s),\ \ \sum_{s \in T} \sum_{i \in s} n_i P_i(s)\,(1 - P_i(s)) \right) \tag{10}$$

Using (9) it is possible to test each partition separately, while (10) provides the overall aggregate test for the entire structure.

<u>The Multinomial-Dirichlet Approach</u>

The Dirichlet probability density function has frequently been used to represent the distribution of the purchase probabilities of consumers in a market. Mathematically, it is given by

$$f(p_1, p_2, \ldots p_b) = \frac{\Gamma(\alpha_1 + \ldots + \alpha_b)}{\Gamma(\alpha_1) \ldots \Gamma(\alpha_b)} \prod_{i=1}^{b} p_i^{\alpha_i - 1}, \qquad 0 < p_i < 1; \quad \alpha_i > 0 \tag{11}$$

where $\sum_{i=1}^{b} p_i = 1$ and $\Gamma(.)$ denotes the gamma function. The density function in equation (11) has (b-1) variates and can be written as

$$f(p_1, p_2, \ldots p_b) = \frac{\Gamma(\alpha)}{\prod_{i=1}^{b} \Gamma(\alpha m_i)} \prod_{i=1}^{b} p_i^{\alpha m_i - 1} \tag{12}$$

where $\alpha = \sum_{i=1}^{b} \alpha_i$ and m_i is the expected market share of brand *i* with $m_i = \alpha_i / \alpha$.

Kalwani and Morrison (1977) suggest that, following the Hendry model, brands are assumed to be directly competing if the switching between them is proportional to their shares of the market, if the market is single partitioned. In a market comprised of multiple partitions, with each partition made up of directly competing brands, switching between brands within each partition will be proportional to their shares within the partition. In the multinomial-Dirichlet model, it may be shown that the expected switching between brands is proportional to their shares within a submarket. That is,

$$P(i, j) = K_w m_i m_j \tag{13}$$

where $K_W = \alpha / (\alpha+1)$ is the switching constant for the focal partition or submarket and m_i and m_j are the shares of brands i and j within the partition. Summing both sides of the equation (13) across all pairs of brands, we find

$$K_W = \sum_{i=1}^{b} \sum_{\substack{j=1 \\ j \neq i}}^{b} P(i, j) / \sum_{i=1}^{b} m_i (1 - m_i). \tag{14}$$

Thus, the switching constant, K_W, is the ratio of actual switching to the switching under homogeneity in consumer brand purchase probabilities when each buyer has probability m_i of buying brand i.

We use equation (14) to estimate K_W for each market partition of the previously identified market partitioning hypotheses. These estimates of the K_W's are then used to compute expected switching levels between brands within and across partitions. The observed switching levels are then compared with the corresponding expected values to determine the goodness-of-fit using chi-square techniques. Interested readers are referred to Kalwani (1984) for the detailed expressions for computing expected switching and repeat purchase levels. Empirical switching levels are then compared with the expected values to determine the goodness-of-fit using chi-square techniques. Revisions in the hypothesized structure are surmised where the theoretical switching levels exceed the empirical switching levels or vice-versa. After one or more iterative attempts, a partitioning structure is identified which provides a reasonably good fit to the brand switching data.

Findings From the French Automobile Market

The brand switching data that we have analyzed pertain to the 198D French automobile market and represent the observed numbers of switchers and repeat purchasers of 15 international brands. The matrix of the observed switching levels is displayed in the appendix. As indicated, first we perform cluster analysis of two different inter-brand substitutability measures to identify tentative market partitioning hypotheses and then use two confirmatory methods to test the fit of the hypothesized competitive market structure.

Figure 1 reveals the findings from cluster analysis of this matrix of inter-brand substitutability measures. As seen in Figure 1, the luxury car brands, BMW, Mercedes, Saab, and Volvo, clearly form a well-defined cluster. The luxury car partition is clearly separated from the other brands. The clustering of the remaining brands is less clear. Ford, GM, Seat, and Volkswagen cluster out of this group simultaneously with the group Fiat, Lada, and Rover. Citroen, Peugeot and Renault join the cluster at the next higher level, and then Alfa-Romeo joins in as a singleton. However, the average linkage distances are so close for these groups that they could very well be in a big cluster of their own, thus forming a market with two submarkets — one made up of four luxury cars and the other with the rest of the brands (see Figure 3a). However, we retain the four way partition of the market for further testing (Figure 3b).

The exploratory analysis of the data with the Rao-Sabavala proximity measures leads to very similar clusters (Figure 2) as the exploratory analysis of brand interaction indices. Once again, the luxury car cluster consisting of BMW, Mercedes, Saab, and Volvo is well-defined. While Alfa-Romeo and Volkswagen do not cluster with any group, clustering among the rest of the brands remains unclear. Fiat, Lada, and Rover cluster first, while Ford, GM, and Seat cluster together. Citroen, Peugeot, and Renault join the clusters at the next higher level. As before, brands other than the four luxury car brands could be grouped together since the separating distances are very small.

The four alternate market partitioning hypotheses that were generated from the cluster analysis of the two brand substitutability measures are displayed in Figure 3. They were subjected to further testing using the "forced switching" approach (UJH 1984) and the multinomial-Dirichlet approach. On applying the UJH confirmatory test to the market with a two-partition structure luxury cars and non-luxury cars in Figure 3a), we found that the z-value for each partition was significant (with p-values < 0.005) and the overall aggregate test was also significant (z-value = 6.14 with p-value < 0.0005), thus rejecting the null-hypothesis of no structure against the alternate hypothesis of a two-partition market structure with luxury cars and non-luxury cars. Next, we applied the UJH procedure to the market structure with four partitions, shown in Figure 3b[1]. While the overall aggregate z-test was significant for this partitioning alternative, we found that the z-value for the Citroen-Peugeot-Renault partition was insignificant, indicating that the brands were not in direct competition. The other market structure with four partitions (see Figure 3c) represents a small modification of the one in Figure 3b. The only change is that Renault has been added to the VW-Ford-GM-Seat partition. We find that the UJH procedure now gives significant test results for all the four partitions and for the overall aggregate test, thus rejecting the null hypothesis in favor of the four-partition market structure shown in Figure 3c.

Our final testing with the UJH approach was for the overlapping structure shown in Figure 3d. This hypothesized market structure is based on the results from cluster analysis of the Rao-Sabavala proximity measure matrix (Figure 2). In this structure, Fiat, Lada, and Rover and Ford, GM, and Seat form separate clusters, but they also overlap among themselves along with Citroen, Peugeot, and Renault. Alfa-Romeo and Volkswagen were not considered as they were singletons. The results show that the UJH procedure rejected the null hypothesis of no structure against the alternative of overlapping structure (see Figure 3d). Next, we applied the multinomial-Dirichlet approach to the structures generated. The findings from this analysis are presented in Table 1. Considering the whole market as a single partition with each brand competing directly against each other, the aggregate χ^2 value was rather high (2179.3) indicating a poor fit and the definite presence of some structure. The fit improved for the two-partition market structure in Figure 3a (χ^2 = 1680.2), but it was still high to give a clear picture of the market. The fit kept improving for the four-partition market structures in Figures 3b and 3c, and was the best for the overlapping structure in Figure 3d. The high χ^2 values in Table 1 are a clear indication that the partitioning in this market is not clear-cut. At the individual partition level, the luxury partition of BMW, Mercedes, Saab and Volvo brands provides a good fit (χ^2 = 17.99 with 12 degrees of freedom, p = .116), implying that the multinomial-Dirichlet model does a good job in representing the observed brand switching patterns within this partition. Finally, the multinomial-Dirichlet model also does a good job in fitting the brand switching patterns within the three-brand partition of Fiat (Italy), Lada (USSR) and Rover (UK). The χ^2 goodness- of-fit measure in this case is 2.18 with 6 degrees of freedom yielding a p-value of .902.

Discussion of Findings

Our purpose in this section is to present our summary view of the structures of the inter-brand competition in the French automobile market Recall that the four competitive structures that we tested using the "forced switching" and multinomial-Dirichlet confirmatory techniques were generated from cluster analysis of two different brand substitutability measures. Each of the four structures performs reasonably well in the "forced switching" approach of UJH. In the

---

[1]Alfa-Romeo was added to the luxury car partitions as the UJH approach cannot handle partitions containing singleton brands.

multinomial-Dirichlet approach, our findings (see Table 1) suggest that multiple partition structures perform substantially better than the single partitioning structure. Also, the two four-partition market structures and the overlapping partition structure yield substantially better results than the two-partition market structure; the differences in fit among the four-partition market structures and the overlapping partition structure, though, are rather small. In sum, based on our empirical results alone, we are not able to-put forth a single, definitive structure of competition in the French automobile market. There are, however, some common patterns of inter-brand competition in the four market structures presented in Figure 3. efore presenting a review of these patterns, we should mention that to supplement our empirical findings, we talked to a number of French students enrolled in the Masters program in Management at Purdue University.

Cluster analysis of brand interaction indices and proximity measures revealed a luxury car partition consisting of Mercedes, BMW, Saab and Volvo. In the UJH approach, this partition is significantly confirmed. Specifically, we find that the Z-value for this partition is 18.21 which is significant at $p < 0.005$ (see Figure 3d). As mentioned, the multinomial-Dirichlet does a good job in representing the observed brand switching patterns among these four brands. Intuitively, the composition of this partition makes sense. All the cars are high-price, luxury cars signifying prestige; the cars are targeted at the same segment of status-conscious affluent customers and, probably, are advertised similarly.

In the three partitioning structures in Figures 3(a), (b), and (c), Alfa Romeo was added to the luxury car partition of Mercedes, BMW, Saab and Volvo. In the UJH approach the expanded submarket of five cars is still significantly confirmed ($Z=16.4$ and $p < 0.0005$). he fit of the multinomial—Dirichlet model, however, is slightly worse ($\chi^2 = 33.87$ with 20 degrees of freedom, $p = .027$). Practically, we find that Alfa Romeo is perceived to be prestigious like the other four cars but has a more sporty image. Therefore, it may be bought as a primary car by a younger population whereas the older customers may buy it as a second car for weekend or pleasure driving.

Three other cars which group together in three of the four partitioning structures displayed in Figure 3 (see panels (a), (b) and (c)) are GM, Ford and VW. A common feature of these three cars is that they are all made in Germany. In our empirical analyses, we find that Seat, which is made in Spain, often ends up being grouped with these three brands. In the UJH approach, Figure 3b reveals that this partition is significantly confirmed with a $z = 13.28$ and $p < .0005$. The multinomial-Dirichlet model provides a poor fit in representing the brand switching patterns within the three-brand partition of GM-Ford-VW ($\chi^2 = 7.77$ with 6 degrees of freedom, $p<0.0005$) and even a worse fit when Seat is included ($\chi^2 = 104.62$ with 12 degrees of freedom, $p<0.00005$).

We find that the three foreign cars, Fiat (Italy), Lada (USSR) and Rover (UK), also tend to group together. A common factor here is that they are all European-made automobiles. They each have relatively small market shares, with Fiat having the largest share among them, viz, 5 to 6 percent. The z-value for this partition is 7.57 and it is significant with a p-value of less than 0.0005. As mentioned, the multinomial-Dirichlet model does a good job in describing brand switching patterns within this partition. The inclusion of Rover, which has a prestigious image, with Fiat and Lada is surprising since the former is a higher quality car which is quite a bit more expensive than the other two.

The final set of three cars, Renault, Citroen and Peugeot, are all made in France and we expected that they would be seen to compete directly and form a partition. The UJH approach yields a z-value of -5.10 which is clearly not greater than 0 and hence, our findings do not suggest that these brands form a partition. The fit of the multinomial-Dirichlet model to the brand switching patterns within this partition is also poor ($\chi^2 = 143.35$ with 6 degrees of freedom, $p < .00005$).

Conclusion

Our analysis has also brought out the advantages and disadvantages of the different procedures that we have used. Clustering procedures are useful and easy to apply as exploratory techniques. However, they are sensitive to the similarity index used in the procedure. Thus, while the two alternative methods we used gave similar results on the luxury car partition, they were different on other aspects. It is essential that great care be taken in developing the similarity index, as clustering is a brute-force method which may force a structure even if there is no underlying structure in the market. However, as mentioned before, it is a useful technique for generating hypotheses about the market.

The UJH procedure is an elegant approach for testing hypothesized market structures. However, the way the test is constructed, with the null of no structure versus an alternate of a *specific* structure, what we test is the fit of the null structure and if it is rejected, we carry forth the alternate structure. Thus, we do not specifically test the fit of the alternate structure that we carry forward. Therefore, it is important to generate many structures and test them. This opens up the possibility of different alternate structures being possible candidates as structures characterizing the market. This is the case in our analysis where the UJH procedure rejects the null in favor of the alternate in almost all cases of partitions analyzed. In such a case, managerial judgment is needed to select the one structure as the best structure.

In the multinomial-Dirichlet approach, on the other hand, we do test the performance of a specific model in fitting brand switching patterns within and across partitions. In our analysis, we found that the overlapping structure had the best fit, making it the best candidate for the structure of the market. However, the procedure also indicates that only in the case of the luxury car partition with Mercedes, BMW, Saab and Volvo, does the model provide an acceptable fit to the brand switching patterns within a partition. The failure of the multinomial-Dirichlet to fit the brand switching patterns in other partitions warrants explanation. One of the reasons for the poor fit may be found in the fact that some brands, specially the market-leading French automobile brands, carry broad product lines under one family brand name. Since they may, therefore, be competing against all other non-luxury cars, the market structure cannot be clearly defined. Empirical support for this explanation is available in the matrix of the deviations of the observed switching levels from the corresponding expected values in computing the overall χ^2 goodness-of-fit measure. These deviations are largest in case of the French cars.

A second reason for the poor fit of the multinomial-Dirichlet model, which assumes stationarity in consumer brand purchase probabilities, may indeed lie in the non-stationarity of the automobile data. There is considerable change in market shares among first and second purchases. This non-stationarity in purchase data may account for at least some of the deviations in observed brand switching levels from the corresponding predicted values based on the multinomial-Dirichlet model.

Simulation studies to test the ability of these different partitioning approaches to recover structure may be a good idea. Such studies may be used to compare their performance across a variety of different cases. However, in conclusion it must be stressed that any analysis of the brand switching data has to be tempered by managerial judgment and additional consumer surveys to increase confidence in the results and the strategies that are based on it.

TABLE 1

Summary of the Goodness-of-Fit Results From the Multinomial-Dirichlet Approach

Hypothesized Structures	χ^2 values
Single partitioned Market	2179.3
Market with 2 partitions (Figure 3a)	1680.2
Market with 4 partitions (Figure 3b)	1595.4
Market with 4 partitions (Figure 3c)	1591.5
Overlapping partition structure (Figure 3d)	1571.5

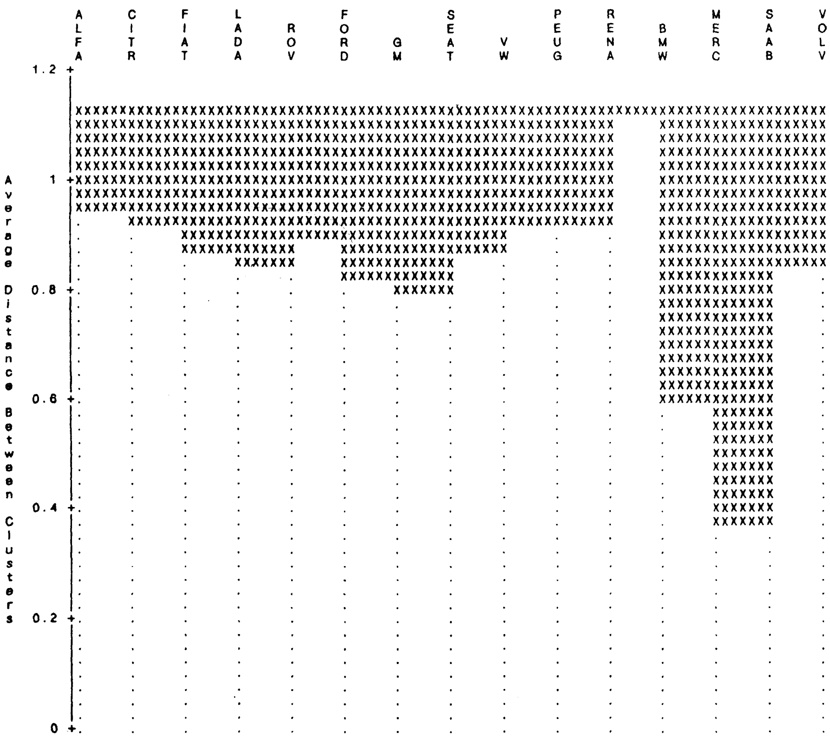


Figure 1. Results from Cluster Analysis of the Matrix of Brand Interaction Indices

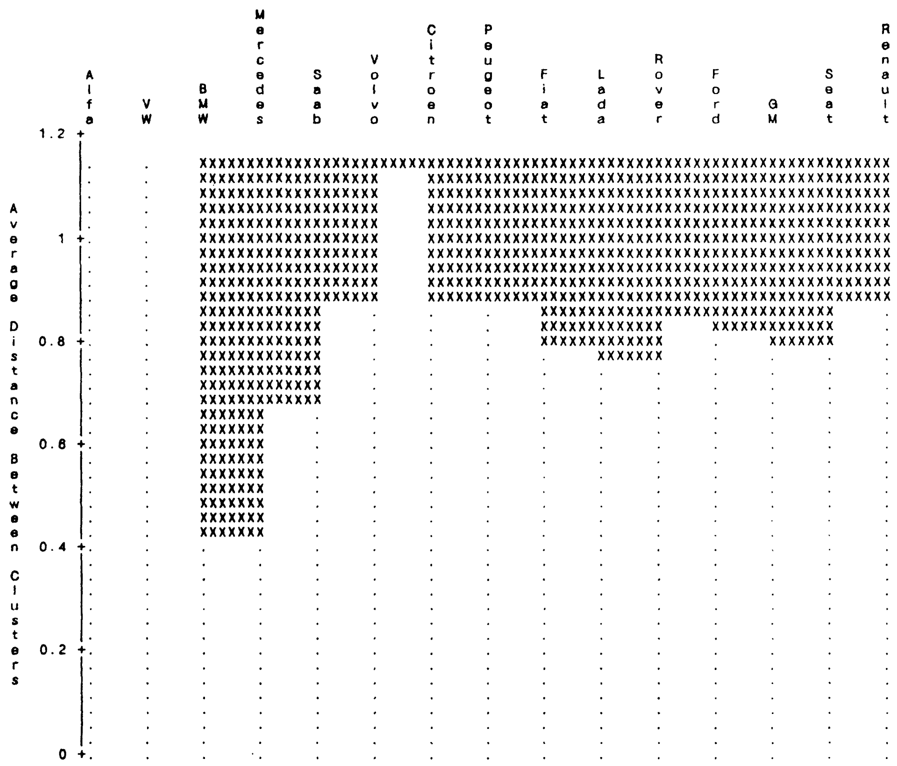
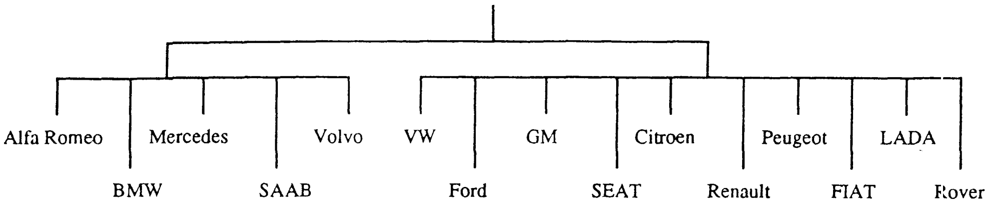


Figure 2. Results From Cluster Analysis of the Matrix of Proximity Measures

(a) 2-way partition

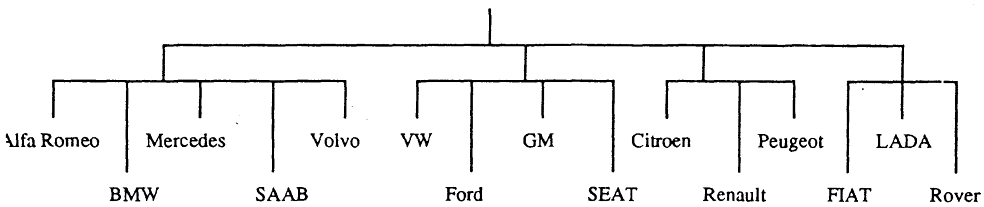


n(s1) = 83
Mean = 16.82
Var = 16.77
Z = 16.4

n(s2) = 8582
Mean =8513
Var = 374
Z = 3.54

Overall Z = 6.14

(b) 4-way partition



n(s1) = 83
Mean = 16.82
Var = 16.77
Z = 16.4

n(s2) = 443
Mean = 250.5
Var = 211
Z = 13.28

n(s3) = 3191
Mean = 3390
Var = 1526
Z = -5.10

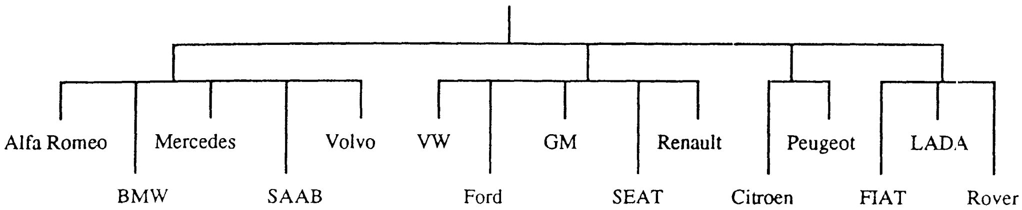
n(s4) = 87
Mean = 40
Var = 38
Z = 7.57

Overall Z = 2.52

Figure 3. Hypothesized structures and results from the UJH approach

Figure 3 (Continued)

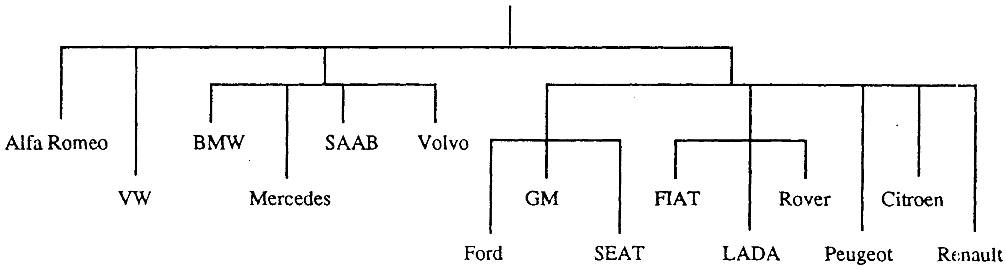
(c) 4-way partition



n(s1) = 83	n(s2) = 1791	n(s3) = 737	n(s4) = 87
Mean = 16.82	Mean = 1634	Mean = 689	Mean = 40
Var = 16.77	Var = 95.28	Var = 540	Var = 38
Z = 16.4	Z = 4.94	Z = 2.15	Z = 7.57

Overall Z = 8.0

(d) overlapping partitions



n(s1) = 60	n(s2) = 182	n(s3) = 87
Mean = 8.27	Mean = 80.69	Mean = 40
Var = 8.07	Var = 73.94	Var = 38
Z = 18.21	Z = 11.78	Z = 7.57

Overall Z = 7.73