

Asymmetry Analysis

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Principal Component Analysis

I apply this to dispersion or correlation matrices. The car data tables do not therefore seem appropriate for principal component analysis. Personally I would much prefer a combination of mover-stayer type modelling and exploratory analyses. In the former one might want to take some account of different car categories. There is an analogy with acoustic confusion - see Morgan (1974), also Copas and Hilton (1990). In the latter case, measures of association will have been formed and analysed by cluster-analyses and scaling methods, by other investigators. The use of metric-scaling is the closest one can come, I think to principal component analysis for these data.

Asymmetry Analysis

I have never attempted an asymmetry analysis before! The following comments are therefore made with some trepidation. The desired analysis follows Constantine & Gower (1978), and is easily carried out in MINITAB. I checked the procedure with the two examples in the above paper. Identical results were obtained in one case, but there were slight differences in the other. In another application the program gave the correct results.

First the rows were scaled to have row total unity, to give matrix \mathbf{M} . The matrix $\mathbf{N} = \mathbf{M} - \mathbf{M}'$ was then obtained. We require the eigenvalues of $-\mathbf{N}^2$ and we plot the cars using the first two eigenvectors. As a general rule, the first pair of eigenvalues seem fairly large, so that the resulting pictures might be expected to provide reasonable descriptions of any asymmetry. In fact the plots seem quite good at drawing attention to large asymmetries but in several cases are rather poor at representing small asymmetries.

As an illustration, I give below the UK8D and UK8C plots, and the FR8D, FR8C and FR8B plots. In each case we determine i) the matrix of asymmetries, \mathbf{N} , ii) the plot, iii) the value of $K9$, which is $K9 = (\lambda^2_1 + \lambda^2_2) / \sum_i \lambda^2_i$, where $\{\lambda^2_{ij}\}$ are the eigenvalues of $-\mathbf{N}^2$, and iv) the $\{\lambda^2_{ij}\}$.

We interpret the plots by forming triangles, one apex at the origin and the other two for any two cars. The area corresponds to the asymmetry. It is striking that Renault occupies an extreme position in all 3 years. To a lesser extent this is also true of Peugeot, and Saab. If we consider Seat for example, Seat drivers change to Renault far more than Renault drivers change to Seat. (By the way, is it possible that the Saab and Seat columns of FR88 are the wrong way around? It certainly looks like it. I have just taken the data as presented.)

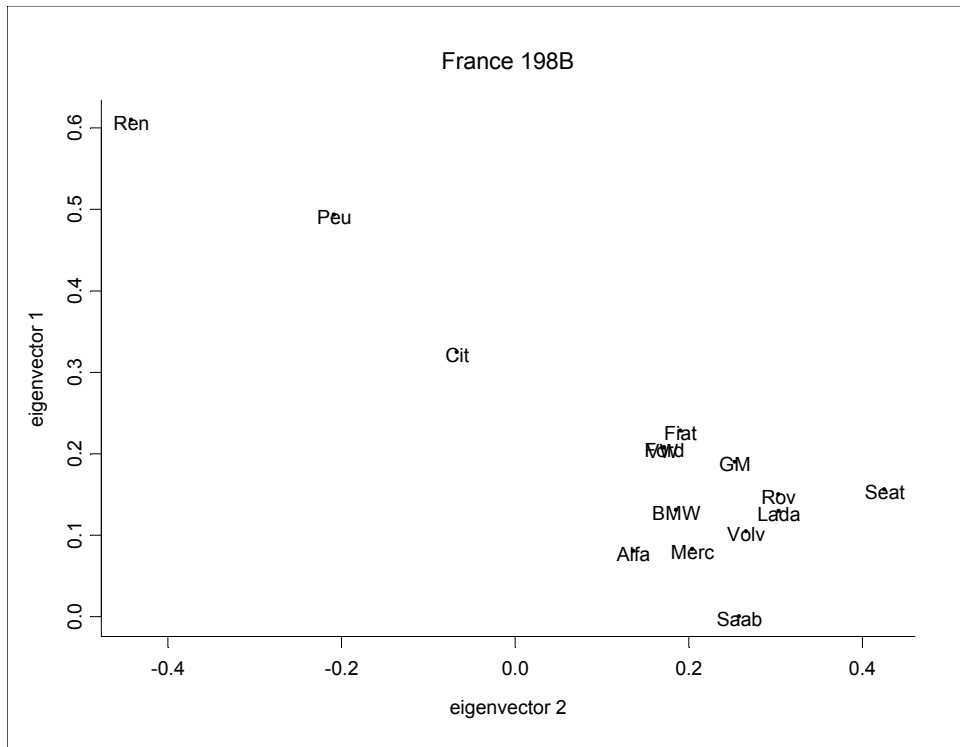
The picture for UK89 indicates the extreme position of Ford, with more (proportions of) drivers changing to Ford (e.g. from Mazda and Porsche) than otherwise.

[Because the graphs and other numeric output reproduced poorly, the analyses were redone in S-Plus. The results were qualitatively similar, but there were some numerical differences – editor]

France 198B

	Alfa	BMW	Cit	Fiat	Ford	GM	Lada	Merc	Peu	Ren	Rov	Saab	Seat	VW	Volv
Alfa	0	-3	1	5	5	0	0	-1	5	13	0	-5	-1	3	-2
BMW	3	0	4	1	3	2	0	-4	12	14	0	-14	-2	4	-1
Cit	-1	-4	0	-10	-5	-8	-15	-9	5	8	-12	-10	-10	-6	-8
Fiat	-5	-1	10	0	0	-1	-1	-2	12	20	-4	-5	-10	-1	-2
Ford	-5	-3	5	0	0	-3	-2	-2	13	19	-4	-5	-3	-2	-3
GM	0	-2	8	1	3	0	-5	-1	13	25	-1	0	-3	2	-3
Lada	0	0	15	1	2	5	0	0	18	21	0	0	0	1	0
Merc	1	4	9	2	2	1	0	0	13	12	0	-4	0	5	-3
Peu	-5	-12	-5	-12	-13	-13	-18	-13	0	9	-16	-10	-24	-11	-15
Ren	-13	-14	-8	-20	-19	-25	-21	-12	-9	0	-24	-14	-32	-18	-19
Rov	0	0	12	4	4	1	0	0	16	24	0	0	-3	2	0
Saab	5	14	10	5	5	0	0	4	10	14	0	0	0	10	5
Seat	1	2	10	10	3	3	0	0	24	32	3	0	0	4	0
VW	-3	-4	6	1	2	-2	-1	-5	11	18	-2	-10	-4	0	-5
Volv	2	1	8	2	3	3	0	3	15	19	0	-5	0	5	0
8693.45		8693.45	309.76	309.76	136.96	136.96	50.16	50.16	25.32						
25.32		5.19	5.19	1.17	1.17	0.00									

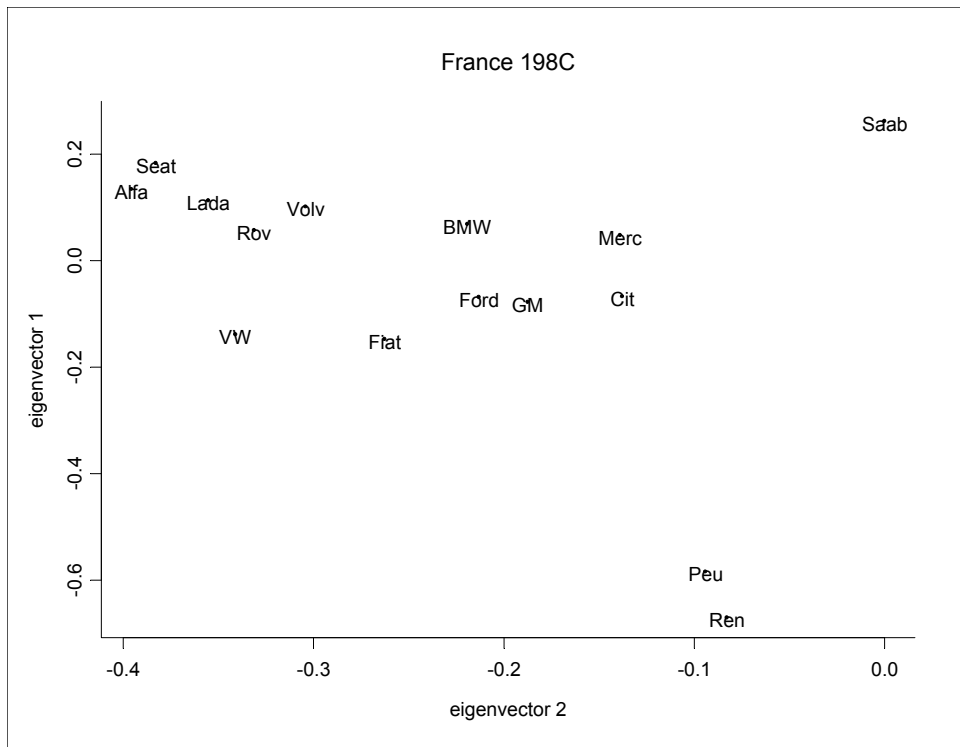
K9 = 0.943



France 198C

	Alfa	BMW	Cit	Fiat	Ford	GM	Lada	Merc	Peu	Ren	Rov	Saab	Seat	VW	Volv	
Alfa	0	1	3	6	3	1	0	1	21	17	1	1	-2	8	0	
BMW	-1	0	0	1	1	2	0	2	9	10	0	-6	1	5	0	
Cit	-3	0	0	-3	0	-1	-2	-1	5	8	-3	1	-4	0	0	
Fiat	-6	-1	3	0	1	1	-11	0	8	11	-7	-4	-4	0	-6	
Ford	-3	-1	0	-1	0	1	-7	-2	12	8	-3	1	-2	0	-1	
GM	-1	-2	1	-1	-1	0	-4	-1	9	9	-8	1	-1	-1	-2	
Lada	0	0	2	11	7	4	0	0	16	18	2	3	-1	-1	-3	
Merc	-1	-2	1	0	2	1	0	0	5	8	1	0	1	4	0	
Peu	-21	-9	-5	-8	-12	-9	-16	-5	0	2	-12	1	-11	-9	-12	
Ren	-17	-10	-8	-11	-8	-9	-18	-8	-2	0	-16	1	-19	-8	-16	
Rov	-1	0	3	7	3	8	-2	-1	12	16	0	-3	-4	0	-1	
Saab	-1	6	-1	4	-1	-1	-3	0	-1	-1	3	0	13	28	5	
Seat	2	-1	4	4	2	1	1	-1	11	19	4	-13	0	15	0	
VW	-8	-5	0	0	0	1	1	-4	9	8	0	-28	-15	0	-2	
Volv	0	0	0	6	1	2	3	0	12	16	1	-5	0	2	0	
4338.0	4338.0	994.1	994.1	105.5	105.5	78.4	78.4	29.0	29.0	15.9						
	15.9	8.0	8.0	0.0												

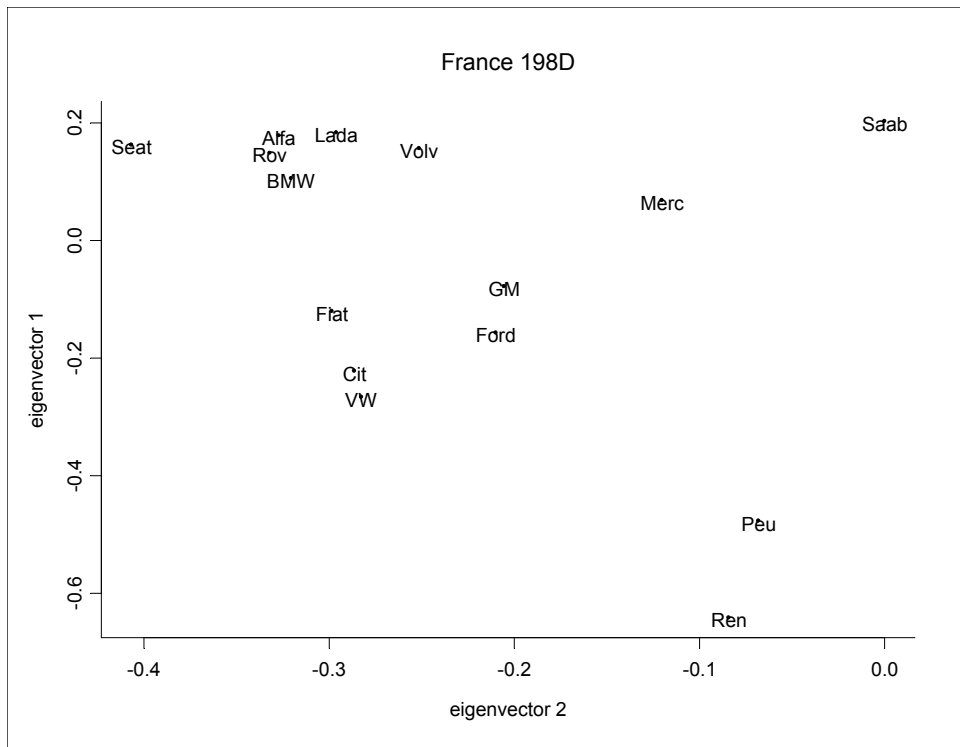
K9 = 0.779



France 198D

	Alfa	BMW	Cit	Fiat	Ford	GM	Lada	Merc	Peu	Ren	Rov	Saab	Seat	VW	Volv
Alfa	0	1	7	6	5	2	0	0	10	13	1	-5	1	6	1
BMW	-1	0	5	2	3	1	0	-3	10	11	0	-10	1	7	-1
Cit	-7	-5	0	-2	-1	-1	-6	-1	9	10	-5	0	-9	-1	-6
Fiat	-6	-2	2	0	0	1	-7	-1	8	9	-5	-5	-6	3	-3
Ford	-5	-3	1	0	0	-1	-6	-2	5	7	-8	0	-5	0	-2
GM	-2	-1	1	-1	1	0	-6	0	6	8	0	0	-6	1	-3
Lada	0	0	6	7	6	6	0	0	14	9	1	0	2	4	0
Merc	0	3	1	1	2	0	0	0	2	5	1	-10	1	2	0
Peu	-10	-10	-9	-8	-5	-6	-14	-2	0	3	-8	0	-6	-9	-5
Ren	-13	-11	-10	-9	-7	-8	-9	-5	-3	0	-13	0	-20	-4	-10
Rov	-1	0	5	5	8	0	-1	-1	8	13	0	0	0	11	-1
Saab	5	10	0	5	0	0	0	10	0	0	0	0	0	10	4
Seat	-1	-1	9	6	5	6	-2	-1	6	20	0	0	0	8	-2
VW	-6	-7	1	-3	0	-1	-4	-2	9	4	-11	-10	-8	0	-10
Volv	-1	1	6	3	2	3	0	0	5	10	1	-4	2	10	0
	3148.93	3148.93	291.68	291.68	136.37	136.37	51.03	51.03	23.45	23.45	9.86	9.86	2.68	2.68	0.00

K9 = 0.859

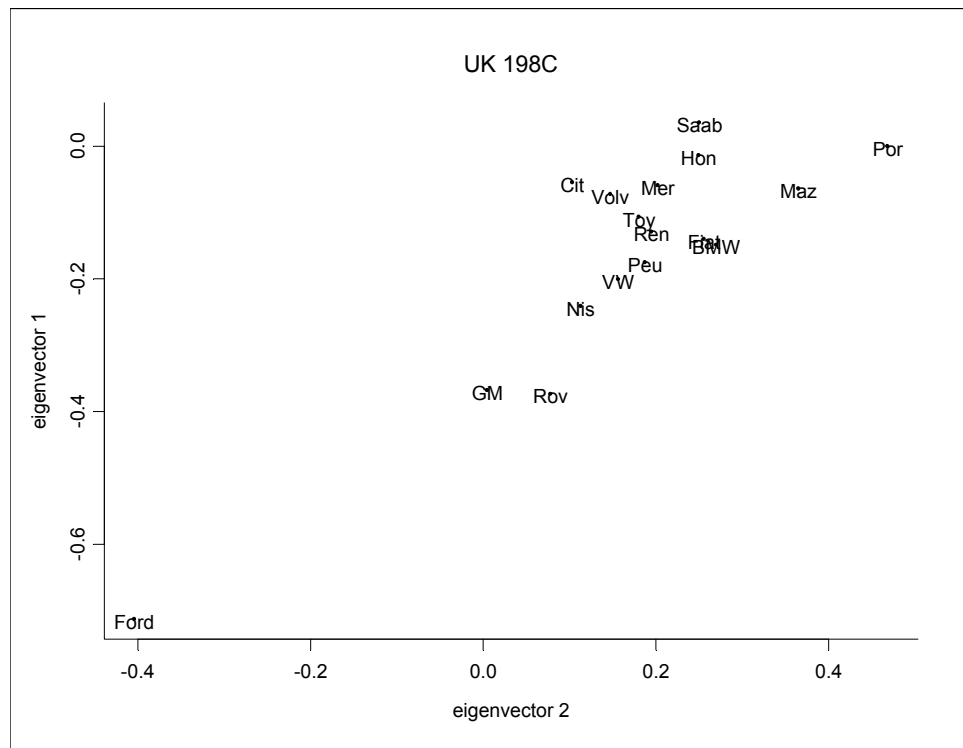


UK 198C

	BMW	Cit	Fiat	Ford	GM	Hon	Maz	Mer	Nis	Peu	Por	Ren	Rov	Saab	Toy	VW	Volv
BMW	0	1	1	12	3	1	-1	-1	0	3	-10	0	6	0	2	4	0
Cit	-1	0	3	4	3	2	-2	-1	2	-1	0	-2	3	-3	-1	0	-1
Fiat	-1	-3	0	13	2	-2	-2	0	4	2	0	0	4	-5	-1	2	0
Ford	-12	-4	-13	0	-8	-9	-17	-9	-9	-10	-18	-9	-11	-5	-6	-8	-7
GM	-3	-3	-2	8	0	-5	-4	-1	-3	-5	-10	-5	-3	-9	-2	-4	-2
Hon	-1	-2	2	9	5	0	1	-1	3	0	0	1	6	1	4	3	2
Maz	1	2	2	17	4	-1	0	0	6	2	0	4	4	1	2	1	0
Mer	1	1	0	9	1	1	0	0	0	2	-3	0	6	0	0	2	2
Nis	0	-2	-4	9	3	-3	-6	0	0	-2	0	-1	2	-8	-8	-2	-1
Peu	-3	1	-2	10	5	0	-2	-2	2	0	-2	-1	5	-3	-3	-3	-1
Por	10	0	0	18	10	0	0	3	0	2	0	2	6	0	8	8	0
Ren	0	2	0	9	5	-1	-4	0	1	1	-2	0	4	-2	-3	0	0
Rov	-6	-3	-4	11	3	-6	-4	-6	-2	-5	-6	-4	0	-4	-3	-2	-4
Saab	0	3	5	5	9	-1	-1	0	8	3	0	2	4	0	0	3	1
Toy	-2	1	1	6	2	-4	-2	0	8	3	-8	3	3	0	0	1	2
VW	-4	0	-2	8	4	-3	-1	-2	2	3	-8	0	2	-3	-1	0	0
Volv	0	1	0	7	2	-2	0	-2	1	1	0	0	4	-1	-2	0	0

2517.8 2517.8 275.8 275.8 93.0 93.0 79.7 79.7 36.9 36.9 14.8
 14.8 4.3 4.3 0.7 0.7 0.0

K9 = 0.833



UK198D

	BMW	Cit	Fiat	Ford	GM	Hon	Maz	Mer	Nis	Peu	Por	Ren	Rov	Saab	Toy	VW	Volv
BMW	0	0	0	6	3	-2	1	-4	-1	4	-13	0	7	-5	1	5	1
Cit	0	0	0	6	4	-1	-2	-1	1	1	0	1	4	0	-1	0	1
Fiat	0	0	0	10	5	-1	-2	0	1	3	0	-1	4	-2	-1	3	0
Ford	-6	-6	-10	0	-8	-5	-7	-6	-3	-7	0	-10	-10	-5	-10	-7	-5
GM	-3	-4	-5	8	0	-4	-4	-2	-2	-1	-13	-3	0	-3	-3	-4	-3
Hon	2	1	1	5	4	0	3	-2	-1	1	0	2	3	-2	6	2	1
Maz	-1	2	2	7	4	-3	0	0	0	7	0	2	6	0	3	1	2
Mer	4	1	0	6	2	2	0	0	0	0	-7	0	4	-3	-1	1	0
Nis	1	-1	-1	3	2	1	0	0	0	-3	0	-3	-1	0	-2	-2	-3
Peu	-4	-1	-3	7	1	-1	-7	0	3	0	0	-3	4	-1	-2	-4	0
Por	13	0	0	0	13	0	0	7	0	0	0	3	3	0	0	2	3
Ren	0	-1	1	10	3	-2	-2	0	3	3	-3	0	4	-3	-3	2	1
Rov	-7	-4	-4	10	0	-3	-6	-4	1	-4	-3	-4	0	-5	-3	-2	-2
Saab	5	0	2	5	3	2	0	3	0	1	0	3	5	0	2	4	6
Toy	-1	1	1	10	3	-6	-3	1	2	2	0	3	3	-2	0	2	3
VW	-5	0	-3	7	4	-2	-1	-1	2	4	-2	-2	2	-4	-2	0	-3
Volv	-1	-1	0	5	3	-1	-2	0	3	0	-3	-1	2	-6	-3	3	0

1521.94 1521.94 304.72 304.72 79.69 79.69 66.78 66.78 30.52
 30.52 11.06 11.06 3.98 3.98 1.31 1.31 0.00

K9 = 0.753

